

CORRIGENDUM TO “WEST’S PROBLEM ON EQUIVARIANT HYPERSPACES AND BANACH-MAZUR COMPACTA”

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In our article [1], on p. 3389, the definition of the weak topology of the G -nerve $\mathcal{N}(\mathcal{U})$ contains a gap. Namely, it is claimed there that the topology on $\mathcal{N}(\mathcal{U})$ induced from \mathcal{J} is the weak one, which is false. The author apologizes for this mistake.

Nevertheless, in the proofs of Lemmas 4.2, 4.4 and 5.2, where the topology of $\mathcal{N}(\mathcal{U})$ is essential, in fact *the right weak topology* of $\mathcal{N}(\mathcal{U})$ was applied. Thus, all of the proofs given in [1] are correct and complete.

Using the notation and references adopted in [1], the above-mentioned gap in the definition of the topology of the G -nerve $\mathcal{N}(\mathcal{U})$ may be filled by replacing the text on p. 3389 starting in line 26 and ending in line 35, by the following.

“For every simplex $L = \langle \mu_0, \dots, \mu_n \rangle \subset \tilde{\mathcal{N}}(\mathcal{U})$, set

$$\Delta(L) = \bigcup \{ \Delta(S, F_S) \mid S \text{ is a subsimplex of } L \}.$$

Clearly, $\Delta(L)$ is an invariant subset of the finite join $G/H_{\mu_0} * \dots * G/H_{\mu_n}$. We always will consider the induced topology and G -action on $\Delta(L)$. Observe that, if N is a subsimplex of L , then $\Delta(N)$ is a closed invariant subset of $\Delta(L)$. Indeed, let $\xi : G/H_{\mu_0} * \dots * G/H_{\mu_n} \rightarrow L$ be the continuous map sending the point $\sum_{i=0}^n t_{\mu_i} g_{\mu_i} H_{\mu_i} \in G/H_{\mu_0} * \dots * G/H_{\mu_n}$ to the point $\sum_{i=0}^n t_{\mu_i} \mu_i \in L$. Since $P_{LN}(F_L) \subset F_N$, where $P_{LN} : \prod_{\mu \in L} G/H_{\mu} \rightarrow \prod_{\mu \in N} G/H_{\mu}$ is the Cartesian projection, we see that the preimage $\xi^{-1}(N)$ is just $\Delta(N)$. Since N is closed in L , this yields that $\Delta(N)$ is closed in $\Delta(L)$, as required. Invariance of $\Delta(N)$ is evident.

It is clear that, if $K \subset \tilde{\mathcal{N}}(\mathcal{U})$ is yet another simplex, then $\Delta(L) \cap \Delta(K) = \Delta(L \cap K)$. Consequently, $\Delta(L) \cap \Delta(K)$ is closed in both $\Delta(L)$ and $\Delta(K)$.

Consider the following invariant subset of \mathcal{J} :

$$\mathcal{N}(\mathcal{U}) = \bigcup \{ \Delta(L) \mid L \in \tilde{\mathcal{N}}(\mathcal{U}) \}.$$

We consider the *weak topology* on $\mathcal{N}(\mathcal{U})$ determined by the family

$$\{ \Delta(L) \mid L \in \tilde{\mathcal{N}}(\mathcal{U}) \}.$$

Namely, a set $U \subset \mathcal{N}(\mathcal{U})$ is, by definition, open in $\mathcal{N}(\mathcal{U})$ if and only if $U \cap \Delta(L)$ is open in $\Delta(L)$ for every simplex $L \subset \tilde{\mathcal{N}}(\mathcal{U})$.

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The G -action on $\mathcal{N}(\mathcal{U})$, defined by the following formula, makes $\mathcal{N}(\mathcal{U})$ a G -space, called the G -nerve of \mathcal{U} :

$$g * \left(\sum_{\mu \in M} t_{\mu} g'_{\mu} H_{\mu} \right) = \sum_{\mu \in M} t_{\mu} g g'_{\mu} H_{\mu}, \quad g \in G.$$

Since the intersection $\Delta(L) \cap \Delta(K)$ is closed in both $\Delta(L)$ and $\Delta(K)$, we see that each space $\Delta(L)$ retains its original topology and is a closed invariant subset of $\mathcal{N}(\mathcal{U})$ (see, e.g., [2, Ch. VI, §8]). We call $\Delta(L)$ a G - n -simplex over the n -simplex L .

In the proofs of Lemmas 4.4 and 5.2 the following well-known and easily proved property of the weak topology is used: a map $f : \mathcal{N}(\mathcal{U}) \rightarrow Z$ is continuous if and only if each restriction $f|_{\Delta(L)}$ is continuous."

As is defined on page 3389, lines 9–10, the elements of a G -normal cover \mathcal{U} are tubular slice-sets gS_{μ} with companion groups H_{μ} . However, in order to emphasize the role of H_{μ} , we have used the denotation (gS_{μ}, H_{μ}) instead of gS_{μ} , which in some occasions may cause confusion. Thus, in Lemmas 4.1, 4.2 and 5.2 the denotation $\mathcal{U} = \{(gS_{\mu}, H_{\mu}) \mid g \in G, \mu \in M\}$ should be replaced by $\mathcal{U} = \{gS_{\mu} \mid g \in G, \mu \in M\}$, where S_{μ} is an H_{μ} -slice.

For the proof of Lemma 5.2 it is important to formulate Lemma 4.1 in the following more precise form.

Lemma 4.1. *Let X be a paracompact G -space and \mathcal{V} an open cover of X . Then X admits a G -normal cover $\mathcal{U} = \{gS_{\lambda} \mid g \in G, \lambda \in \Lambda\}$ with the companion groups $\{H_{\lambda}\}_{\lambda \in \Lambda}$ such that each H_{λ} is the stabilizer of a point $x_{\lambda} \in S_{\lambda}$ and \mathcal{U} is a star-refinement of \mathcal{V} .*

In Lemma 5.2 under the term " ε -cover" we mean the family of *all* open balls in $L_0(n)$ which have radius ε .

In the formulation and in the proof of Lemma 5.2, always $G = O(n)$.

Also, one should correct the following misprints:

- (1) page 3389, line 14: " $O \in U_1$ " should be " $O \in \mathcal{U}_1$ ".
- (2) page 3389, line 20: "of \mathcal{U} " should be "of \mathcal{V} ".
- (3) page 3390, line 27: " (gS_x, G_x) " should be " gS_x ".
- (4) page 3390, line 30: "an open G -normal cover" should be "a G -normal cover".
- (5) page 3391, line 28: " $\Delta(L, F_L)$ " should be " $\Delta(L)$ ".
- (6) page 3392, line 2: " $R(x) \in \mathcal{F}_{3^{n-1}}(s)$ " should be " $R(x) \in \mathcal{F}_{3^{n-1}}(s^1)$, where s^1 is the 1-dimensional skeleton of s ".
- (7) page 3392, line 7: " $\Delta(L, F_L)$ " should be " $\Delta(L)$ ".
- (8) page 3392, line 13: " $\mathcal{F}_{3^{n-1}}(s) \subset$ " should be " $\mathcal{F}_{3^{n-1}}(s^1) \subset$ ".
- (9) page 3394, line 30: " $q'(g_1 A_{\mu}) = q''(g_0 A_{\lambda})$ " should be " $q'(g_1 H_{\mu}) = q''(g_0 H_{\lambda})$ ".
- (10) page 3394, line 36: "of $g_0 A_{\lambda}$ " should be "of $g_0 A_{\lambda} \cup g_1 A_{\mu}$ ".
- (11) page 3394, line 41: "of $g_1 A_{\mu}$ " should be "of $g_0 A_{\lambda} \cup g_1 A_{\mu}$ ".
- (12) page 3398, line 22: "domain. Since" should be "domain V . Since".

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